

Inference at *
of proof for Lemma exists_functionality_wrt_implies:

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⊢∀S,T:Type, P,Q:(S→ ℙ).  
  (S = T) ⇒ (∀x:S. {P(x) ⇒ Q(x)}) ⇒ {(∃x:S. P(x)} ⇒ (∃y:T. Q(y))}  
  by (((Unfold 'guard' 0)  
  CollapseTHEN (UnivCD)).)  
  CollapseTHENA (  
    (Auto_aux (first_nat 1:n) ((first_nat 1:n),(first_nat 3:n)) (first_tok :t) inil_term)))
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1:

1. $S : \text{Type}$
2. $T : \text{Type}$
3. $P : S \rightarrow \mathbb{P}$
4. $Q : S \rightarrow \mathbb{P}$
5. $S = T$
6. $\forall x:S. P(x) \Rightarrow Q(x)$
7. $\exists x:S. P(x)$
- $\vdash \exists y:T. Q(y)$